Basic Coaxial Mass Driver Reference Design *

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Abstract

The reference design for a basic coaxial mass driver is developed to illustrate the principles and optimization procedures on the basis of numerical integration by programmable pocket calculators. The four inch caliber system uses a single-coil bucket and a single-phase propulsion track with discrete coils, separately energized by capacitors. An actual driver would use multiple-coil buckets and an oscillatory multi-phase drive system. Even the basic, table-top demonstration system should in principle be able to achieve accelerations in the 1,000 m/s^2 range. Current densities of the order of 25 kA/cm^2, continuously achievable only in superconductors, are carried by an ordinary aluminum bucket coil for a short period in order to demonstrate the calculated acceleration. Bucket current is supplied through contacts sliding along tubular guide rails. Ultimately the system can be lengthened and provided with a magnetically levitated, superconducting bucket to study levitation dynamics under quasi-steady-state conditions, and to approach lunar escape velocity in an evacuated tube.

1. Introduction

The direct use of electromagnetic energy for accelerating macroscopic rather than sub-atomic matter is an old dream which inspired several premature and dramatically unsuccessful attempts over the years. An electromagnetic cannon built in Germany during World War II, based on a linear induction motor, tended to melt its projectiles, and an aircraft launcher built in the U.S. during the forties, the "Westinghouse Electropult", based on a mechanically commutated linear dc motor, failed to come even close to the performance of steam and compressed air devices.

Electromagnetic flight and rocketry has become practical only recently with the advent of superconductors capable of carrying loss-less persistent currents about 2,000 times higher than can be sustained in ordinary conductors of the same cross section. Superconducting coils energized with persistent current now provide us with large magnetic dipoles, such as could previously be generated only at prohibitive cost of weight and power. Powell and Danby[1] of the Brookhaven National Laboratory first pointed out in 1966 the possibility of magnetically levitated and propelled high speed ground transportation based on vehicles carrying superconducting dipoles. The proposal stimulated a substantial international research effort, which is reviewed by Chilton at the present conference[2]. An earlier review by Thornton[3] cites 124 references. O'Neill first called attention to the space application of electromagnetic rocketry in 1974 [4,5], recognizing the enormous acceleration which is achievable by synchronous propulsion. A 1975 article by Kolm and Thornton[6] suggested to him the last remaining element needed: a means for resiliently supporting vehicles at essentially unlimited speed. In the summer of 1976, O'Neill organized and directed a collaborative effort at the NASA-Ames Research Center which resulted in the first in-depth study of electromagnetic mass drivers, namely devices for accelerating mass by means of recirculating shuttle buckets with superconducting coils, levitated, guided and propelled by a travelling magnetic field [7,8,9].

Two basic configurations were investigated in the Ames study: a planar one, combining the guidance geometry used earlier in a rocket sled design by Chilton et al[10], and the synchronous propulsion System used in the M.I.T. Magneplane[11], for which extensive computation had been done by a Raytheon group[12], and a coaxial configuration proposed by me which had not previously been considered.

Following the Ames study, Prof. O'Neill came to M.I.T. for the academic year as Jerome Hunsaker Visiting Professor or Aeronautics and Astronautics before returning to the Physics Department at Princeton University. This provided an opportunity for us to collaborate in a continuation of the mass driver study. The work is being conducted as a student project with modest support from the M.I.T. Sloan Fund, the Department of Aeronautics and Astronautics, the Francis Bitter National Magnet Laboratory (sponsored by the N.S.F.), and NASA-Ames. The student group is headed by Kevin Fine, a master degree candidate in the Department of Aeronautics and Astronautics; the group also includes William Wheaton, a post-doctoral fellow, and students Eric Drexler, William Snow, Jonah Garbus, and Jon Newman.

A further comparison of the planar and coaxial configurations soon revealed a distinct superiority of the coaxial one, at least for reaction engine applications where total system mass is more crucial, and very probably also for the lunar launcher application. This superiority stems from the fact that inductive coupling between bucket and propulsion coils is inherently tighter, which permits more effective utilization of conductor mass, and both the bucket and the guideway structures are subjected only to pure tension forces, which permits higher acceleration before structural limits are reached.

It was therefore decided to concentrate on the coaxial configuration and to design and construct what can be called a "basic" system, one which is simpler than a high-performance system for space use and still involves all the basic features and design problems relating to propulsion. Only the

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levitation and guidance problems are bypassed in the initial version. Basic though it may be, the model in question is not a mere toy. It should accelerate a half-kilogram vehicle to 100 gravities, or 1,000 m/s^2. This means that the vehicle would emergp from a 2 m accelerating section at 160 mph, with an energy of 1,000 joule.

The study I shall describe serves several distinct functions. It will clarify the basic principles and problems involved in designing, optimizing and building mass drivers. It will demonstrate the potential capabilities and limitations of mass drivers in relation to various possible applications and missions. It will point toward the logical direction of future research and development efforts. It will train the first team of experts in this new field, and lay the groundword for the first course and textbook.

2. Definition of Basic Mass Driver

The "basic" mass driver studied here differs from an actual space device in the following essential respects:

The bucket (which might be called the "shuttle", if the term had not acquired another connotation) carries only one single coil instead of a series of probably four coils of alternating polarity.

This single coil is made of aluminum rather than superconductor, and is supplied with direct current through a set of eight carbon brushes sliding along four copper tubes. The current density is comparable to values achievable in a superconducting coil, but can only be maintained for a fraction of a second.

The bucket is guided by the four current supply tubes instead of being guided by repulsion forces exerted by eddy currents induced by the bucket's motion in surrounding aluminum guide surfaces, without physical contact.

Propulsion coils are energized individually by separate electrolytic capacitors, discharged by individual SCRs when the bucket reaches a predetermined position, and crowbarred by diodes to ensure unidirectional (non-reversing) current. An actual space system would use a multi-phase propulsion coil array, energized in groups by oscillatory circuits, in the manner described in the Ames study report[8,9].

The basic driver operates on a single-shot basis rather than launching vehicles in rapid succession.

The basic driver operation is limited to a fraction of a second by temperature rise in the bucket coil.

A logical second version of the basic driver would use a superconducting bucket coil and repulsive guide surfaces, and could be operated in a race-track loop having sufficient straight length to permit studying the levitation dynamics under quasi-steady-state conditions.

3. Choice of Dimensions

The optimum size of a mass driver is governed by the optimum acceleration for the intended mission. Acceleration imposes an upper limit on size because of the invariance of material strength.

The lower limit of size is imposed by irreducible clearance required between bucket and drive coils. This gap must accommodate guide rails of adequate thickness and strength, and it must accommodate inevitable mis-alignment and oscillations of the bucket. The gap must also accommodate a radiation shield, and in the case of terrestrial operation a vacuum insulation jacket around the superconducting coils. We selected a nominal caliber of four inches, which is smaller than that of an optimized lunar mass driver and larger than that of an optimized, precisely constructed reaction engine intended for an asteroid retrieval mission. It represents a logical compromise between excessive cost and excessively severe precision requirements.

Fig. 2 shows the dimensions which follow quite logically once a caliber of four inches has been chosen. It was decided to use 20 drive coils spaced 10 cm center to center, all but the first three having the cross section shown. The first three coils have enlarged cross sections for reasons to be discussed later. The number of turns of each coil was also adapted to the bucket speed, on the basis of electrical considerations to be discussed. Fig. 3 is a section view of the bucket in its second generation version (the first was needlessly heavy). Details concerning the construction of the model are given in a companion paper by Fine[13].

4. Thrust Calculation

Lorentz Force Method

All forces on the bucket, thrust force as well as lateral forces, are the sum of elementary Lorentz forces acting on every element of current-carrying conductor. The elementary local force vector dF is the cross product of the local current vector and the local magnetic field vector dB. The current vector is the element of conductor length ds, times the total bucket current it conducts, I_b:

$$\vec{dF} = I_b \, \vec{ds} \times \vec{B} \qquad (1)$$

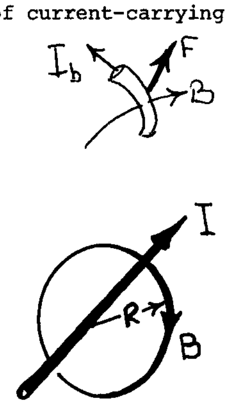

Fig. 1: The Lorentz force and the Biot-Savart Law for approximate calculation of forces.

The off-axis magnetic field generated by a coil can only be computed by the use of finite elliptic integrals, or directly by numerical summation of current elements. The Lorentz force method of force calculation is therefore useful only if a computer and software for off-axis field computation are available, or if only a crude estimate of the force is required. For such estimates it is useful to recall the Biot-Savart Law, which states that the magnetic field near a long, straight conductor is tangential with respect to the conductor and has the intensity

$$B = \frac{\mu_o I}{\pi 2 \ R} \qquad (2)$$

in rationalized MKS or SI units, where R is the distance of the field point from the wire, and is small compared to the length of the wire.

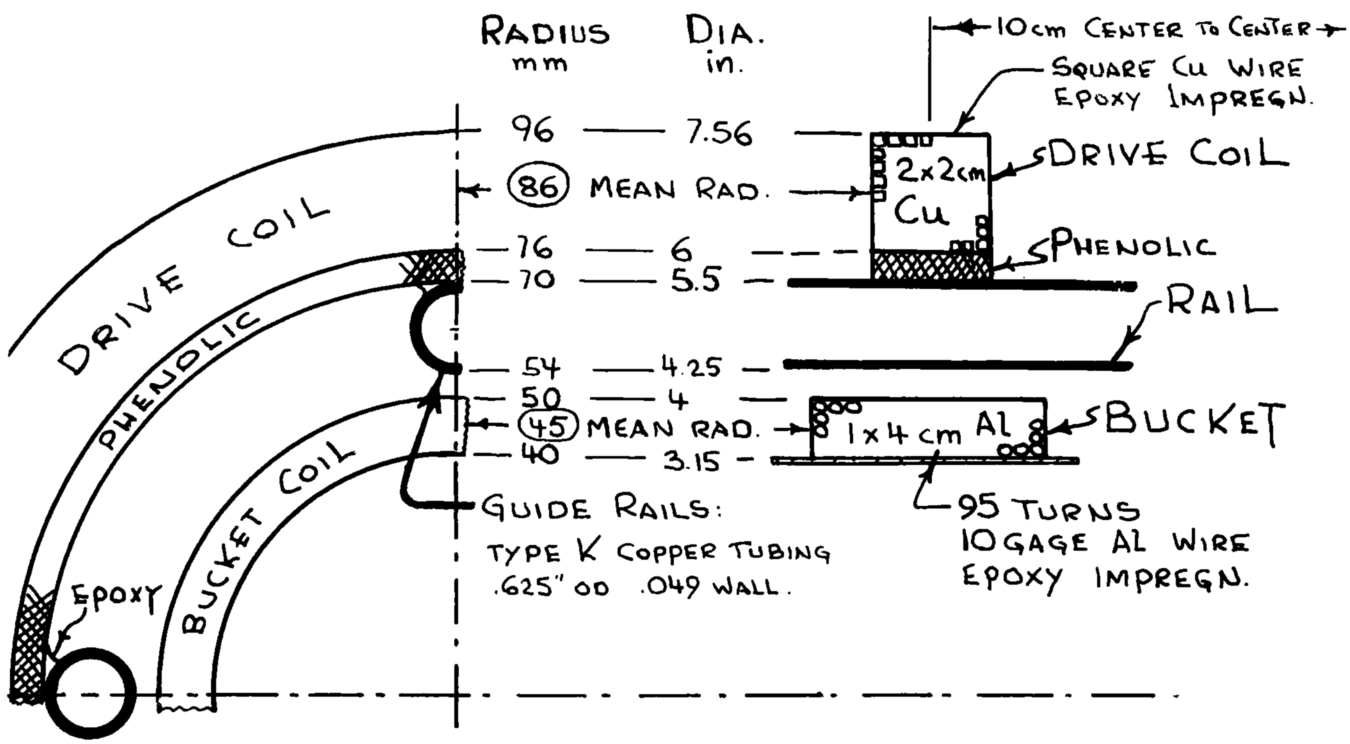

Fig. 2. Dimensions of the basic four inch caliber coaxial mass driver

The Lorentz force is also conceptually useful. It makes obvious the fact, for example, that only the radial component of magnetic field provides thrust, while the longitudinal component only generates lateral force on the bucket coil.

Inductance Method

A more directly useful method of force calculation, and one which provides topological insight, is based on the fact that force is simply the rate of change of stored energy with motion, or the energy gradient in the direction of motion. The energy stored in a system of current-carrying conductors is simply

$$\frac{1}{2} L I^2 \quad , \tag{3}$$

where L is inductance. It is best to think of inductance as the magnetic flux linking a circuit per unit current in that circuit. In the system comprising a bucket coil and the nearest drive coil, inductance includes three terms: the self-inductance of each coil, and the mutual inductance between them, which is the flux linking either one of the coils due to unit current in the other, and this energy term enters twice. The total magnetic energy stored in the system is thus given by:

$$E = \frac{1}{2} L_b I_b^2 + \frac{1}{2} L_d I_d^2 + M I_b I_d \tag{4}$$

where the subscripts b and d refer to the bucket and drive coil respectively, and where M denotes mutual inductance. It is clear that motion of the bucket coil will change only the mutual inductance term of the stored magnetic energy, and the rate of change of energy with motion is therefore simply

$$\frac{dE}{dx} = \frac{dM}{dx} I_b I_d = F_x \tag{5}$$

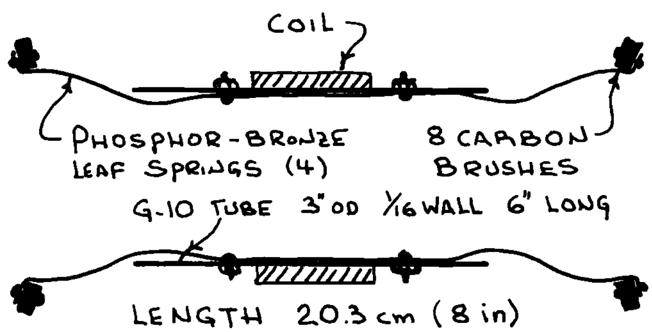

Fig. 3. Section view of half kilogram bucket

where F_x represents the force in the same direction in which the mutual inductance gradient is taken. Thus, to obtain the force acting on a bucket coil in any direction, it is merely necessary to measure or calculate the mutual inductance between that bucket coil and all relevant drive coils as a function of position, and then to derive the gradient of mutual inductance along the desired direction. It is very easy to explore the thrust and restoring force profiles of any mass driver configuration by constructing a mock-up system of bent wire and aluminum sheet, and then plotting the mutual inductance as measured with a bridge at suitable frequency as a function of bucket position. This "inductance simulation technique" was developed and used success-

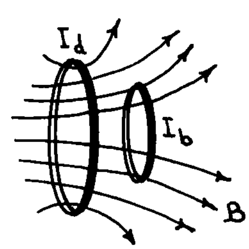

Fig. 4. Mutual inductance is flux linkage.

fully by Iwasa[14] to predict the restoring force and torque profiles of the magneplane vehicle in its cylindrical guideway trough.

Self and mutual inductances of a variety of coil configurations can also be obtained from a classical tabulation by Grover[15] based on elliptic integral computation. The work is unfortunately out of print, but is available in most engineering libraries. We have obtained the relevant inductance values for the basic coaxial driver configuration shown in Fig. 1, with drive coils spaced 10 cm center-to-center. Self-inductances are based on current distributed over actual cross sections of each coil, while mutual inductances assume the current to be concentrated in a filament located at the mean diameter of each coil. This approximation introduces an error of 2% in the mutual inductance between neighboring drive coils, and probably a somewhat larger error in the mutual inductance between the drive coil and the elongated bucket coil when it is very near the drive coil.

All inductance and resistance values listed below refer to coils having dimensions shown in Fig. 2, and a single electrical turn. To obtain corresponding values for coils having the same cross section but divided into N turns, the single-turn values must be multiplied by N^2:

$$L_N = L_o N^2; \quad M_N = M_o N_1 N_2; \quad R_N = R_o N^2 \tag{6}$$

The mutual inductance gradient dM/dx is also shown graphically in Fig. 5, along with an analytic approximation curve fitted by O'Neill[16].

bucket coil, aluminum, 20°C	L_o = 0.0874 μH, R_o = 19.8 μΩ
drive coil, copper, 20°C	L_o = 0.254 μH, R_o = 24.3 μΩ
neighboring drive coils 10 cm spacing	M_o = 0.0338 μH; $\frac{dM}{dx}$ = 0.535 μH/m

bucket to drive coils:

x (position) (m)	dM/dx (μH/m)	M (μH)
0 (centered)	0	.0523
.01	.1648	
.02	.4379	
.03	.5741	
.04	.5965	
.05	.5517	
.06	.4796	
.07	.4043	
.08	.3353	
.09	.2755	
.10	.2259	.0118
.11	.1846	
.12	.1515	
.13	.1247	
.14	.1033	
.15	.0857	
.16	.0714	
.17	.0598	
.18	.0507	
.19	.0427	
.20	.0341	.0027

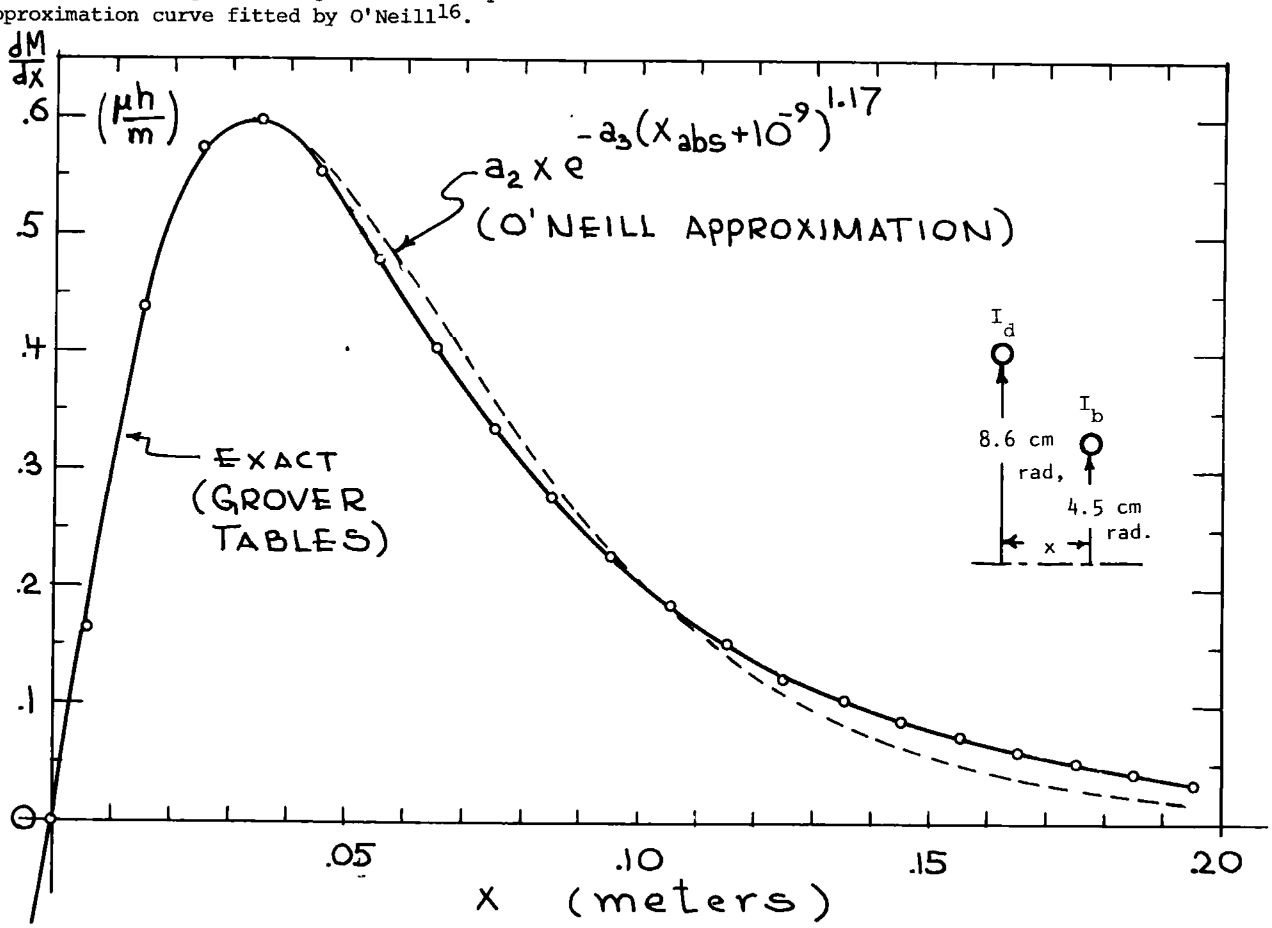

Fig. 5. The mutual inductance gradient dM/dx as a function of bucket position x.

In order to calculate the thrust force F from equation (5) we need two additional quantities: the dc current in the bucket coil, and the time-varying current in the drive coil. The object of our calculation will be to optimize the time-variation of the drive current.

5. Bucket Coil Design

An operational mass driver for space would have several bucket coils of superconducting cable, energized with persistent current at a density of about 25,000 A/cm^2 of cross section. As a basis for optimization and scale-up calculations, I cite the actual performance achieved in about a hundred test runs of the MIT Magneplane model, built with materials available in 1973: the current density, referred to total cable cross section, was 25,000 A/cm^2, and the decay constant of the current was 400 hours. A typical cable is composed of aluminum, density 2.8, and niobium-titanium, density 8.5, in the ratio of 2.3 to 1, resulting in a total density of 4.53 g/cm^3; (see ref. 11). The cable would be composed of a large number of twisted and transposed filaments, cross laid (like lightning conductor cable) to achieve about 30% to 50% void space, and swaged into a stainless steel envelope. Cooling would be accomplished by a permanently sealed charge of helium, refrigerated at periodic intervals by contact with a heat sink. The current would be adjusted inductively during this re-cooling cycle. The cable envelope would withstand the helium pressure even if the bucket were to be heated in direct sunlight.

The basic driver has a single aluminum coil, energized through automobile starter brushes sliding along copper guide rails, as mentioned earlier, and described in detail in the companion paper by Fine. It has a mean radius of 4.5 cm, a length of 4 cm, and a build of 1 cm, as shown in Fig. 2. Wound with aluminum of density 2.8, neglecting void space, the coil has a mass of 305 g, a single-turn resistance at 20°C of 19.8 μΩ, and a single-turn inductance of 0.0874 μH.

To achieve an acceleration in the 1,000 m/s^2 range, we need a bucket current of about 50 kA, (half the current density of a superconductor), for a period of about 0.126 s (assuming 2 m of acceleration and 2 m of deceleration). the I^2R heat input is 49.5 kW or 6.26 kJ of total energy. At a specific heat of 0.84 J/g, the temperature in the aluminum will rise about 24.4°C. For switching considerations it is also of interest to calculate the magnetically stored energy: $1/2\ L\ I^2 = 109$ J.

It now remains to divide the winding into a practical number of turns. Formvar-insulated 10-gage aluminum wire has a nominal diameter of 0.102" bare and 0.105" insulated, with a 20°C resistance of 1.64 ohm/1000 ft and a weight of 9.55 lb/1000ft. The winding space allocated (Fig. 2) will accommodate five layers of 19 stack-wound turns, or 95 turns altogether. This amounts to 87 ft weighing 376 g and having a resistance which rises from 0.14 ohm at 20°C to 0.15 ohm at 45°C.

For 50,000 ampere turns, the coil requires 526 A at 74 V. This can be obtained readily from fourteen 12 V auto batteries, connected 2 in parallel and 7 in series. This massive energy storage requirement represents the penalty for not using a superconducting bucket coil. None of this energy goes into propulsion. The value of superconductivity is underscored by the ironic fact that it replaces man's only device which operates at zero efficiency.

Cooling the bucket coil to 77°K with liquid nitrogen will reduce the resistivity of the aluminum by a factor of four (possibly more if it is pure and annealed, and not excessively work-hardened during winding). With an initial coil resistance of only 0.035 ohm, the voltage required drops to 18V. The average specific heat of aluminum in the 77 - 100 degree range is 0.5 J/g deg. This heat input is thus reduced to 9.7 kW, and the temperature rise to about 10°C. The energy can now be supplied by only four 12 V auto batteries, two in parallel and two in series.

6. Basic Propulsion System

Propulsion is accomplished by transforming energy stored as charge on a capacitor into energy stored as current in an inductor, the drive coil, and then converting as much as possible of this energy into kinetic energy of the bucket. The energy transfer from capacitor to inductor is highly underdamped, meaning that only a small fraction of the stored energy is lost in the transfer, either as heat in the resistance of the circuit or as kinetic energy absorbed (usefully) by the bucket. If left to itself, the discharge would therefore oscillate or "ring" with gradually diminishing amplitude, until all of the energy has been dissipated.

In an operational, full-wave mass driver the discharge would be allowed to oscillate through one complete cycle by means of a circuit described in the Ames 76 study report[8]. Energy would thus be imparted to the bucket during both its approach and departure from each drive coil, and this process permits energy transfer, from capacitive to kinetic, at efficiencies as high as 96%. The circuit also permits effective regenerative braking, with recovery of about 80% of the kinetic energy.

The basic mass driver, however, is based on electrolytic capacitors such as are used, for example, to store energy for electronic photoflash devices. Such capacitors are very economical in mass and cost per joule of stored energy, but their electrolytically formed insulation would be destroyed by a voltage reversal. The discharge must therefore be short-circuited or "crowbarred" at the instant the capacitor voltage reaches zero. Instead of returning to the capacitor, the current therefore continues to flow through the short-circuited coil until it has dissipated all of its energy in the circuit resistance, long after the bucket has departed. The simplicity of the basic half-wave system is purchased at a considerable cost in efficiency: only about one third of the capacitor energy is converted into kinetic energy of the bucket. The system does however illustrate the design principles and demonstrate the performance as well as a more sophisticated mass driver would.

The propulsion circuit, along with its current and voltage waveforms, is shown in Fig. 6. It operates in the following manner. When the bucket reaches a pre-determined position (optimally about

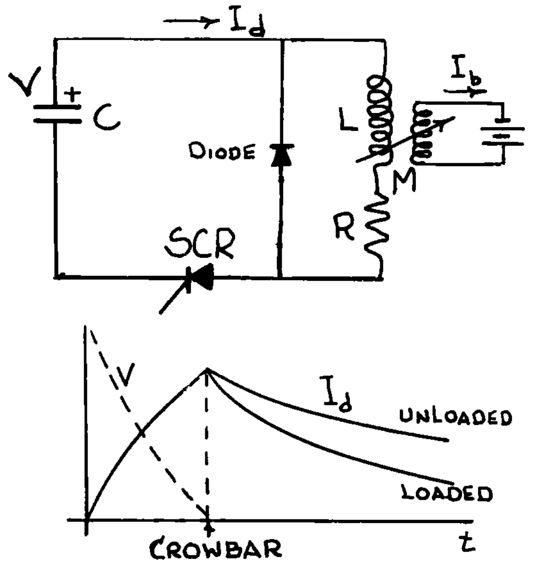

Fig. 6. Basic propulsion circuit and waveforms

1 cm before dead center), the silicon controlled rectifier (SCR) is triggered and permits current to flow from the capacitor C through the drive coil indicated by L and R. The current I_d rises and the capacitor voltage drops, in the manner shown. When the capacitor voltage reaches zero and begins to reverse, the diode starts to conduct in its forward direction, thereby preventing the current from returning reverse charge into the capacitor by allowing it to flow through the "crowbarred" coil. The current after crowbarring will thus decay. In the absence of the bucket (unloaded condition), this decay is governed exclusively by the inductance/resistance ratio, the L/R time constant, of the drive coil, and will follow the familiar exponential decay:

$$I_t = I_o \; e^{\frac{-t}{L/R}} \qquad (7)$$

This L/R time constant depends only on the cross-section area of the drive coil, and does not depend on the number of turns, since both L and R are proportional to N^2 (eq. 6). In the presence of the bucket (loaded condition), the current will decay more quickly because additional energy is extracted by the bucket.

The bucket is indicated in the circuit diagram by a coil through which direct current is driven by a battery, and which is inductively coupled to the drive coil by a variable mutual inductance M. This mutual inductance depends on the position of the bucket coil, and its variation dM/dx is shown in Fig. 5.

We now wish to compute the performance of the propulsion system in a manner which preserves intuitive insight into the fundamental relations involved in designing and optimizing more sophisticated mass drivers. To do so, we neglect two effects which, though small, are not quite negligible.

We assume that the bucket current I_b is constant, neglecting the fact that the changing magnetic flux linkage actually makes I_b increase slightly. The effect will be significant only at the instant the instant the bucket coil passes through the center of the drive coil when mutual inductance is greatest and the drive coil current I_d is changing most rapidly, particularly at the high velocity end of the track. Another way to express this effect is to say that the system would accelerate as a linear induction motor even if the bucket coil were to be short-circuited without a current source, like a squirrel-cage induction rotor, and that this induction effect would supplement the synchronous acceleration which dominates. The approximation is thus a conservative one, except for the fact that the SCR and diode current will be slightly higher.

The second approximation we make is to calculate the interaction between the bucket and each drive coil, while the bucket moves from this coil to the next one, as if no other drive coils existed, and as if the propulsion force vanished abruptly as soon as the bucket reaches the next coil. Actually, each drive coil continues to accelerate the bucket even after it passes beyond the next drive coil. Each drive coil also induces some current, at the expense of its own current, in the two nearest neighbor coils whenever they are not open-circuited by their SCRs. In other words, each drive coil is loaded slightly by its predecessor and by its successor at the time the successor is triggered.

With these two approximations, circuit analysis becomes straightforward. Voltage drop around the loop must be zero. In terms of the instantaneous current i, this can be expressed by:

$$V - L_o N^2 \frac{di}{dt} - N \, I_b \frac{dM}{dt} - i \, R = 0 \qquad (8)$$

The capacitor voltage will decrease as charge is drained from it at the rate i:

$$\frac{dV}{dt} = \frac{i}{C} \qquad (9)$$

and the rate of mutual inductance change is

$$\frac{dM}{dt} = \frac{dM}{dx}\frac{dx}{dt} = \frac{dM}{dx} u \text{ (u=velocity)} \qquad (10)$$

velocity u can be related to the current i through the expression we have derived for the thrust force F_x (Eq. 5), since F_x = mass x acceleration:

$$\text{acceleration} = \frac{du}{dt} = \frac{F_x}{m} = \frac{I_b N}{m} \frac{dM}{dx} i \qquad (11)$$

Since dM/dx has already been determined (Fig. 5), we are now able to calculate the motion of the bucket as it passes through the mass driver from coil to coil.

The only parameters not as yet specified are the capacitors and the number of turns for the drive coils.

A logical design decision is to use identical capacitors at the highest voltage rating available in order to minimize cost per joule and also SCR current rating, and then to optimize the coil turns on the basis of this choice.

A convenient length for the accelerating section is 2 meters, and at our selected coil spacing of 10 cm, this will accommodate 20 drive coils. We shall energize each coil from an electrolytic capacitor rated at 450 V, 2,000 μF; the stored energy will be $1/2\ C\ V^2$ = 203 joules.

With this choice, the number N of turns in each drive coil will govern both the maximum current and the rate of current rise in the coil before crowbarring occurs. It will be recalled that the rate of current decay after crowbarring depends only on the cross section of each coil. The free, uncrobarred discharge would ring in the manner indicated in Fig. 7, at a frequency given by

$$\omega = \frac{1}{LC} = \frac{1}{N}\ \frac{1}{L_o C} \qquad (12)$$

radians/second, and the maximum current is obtained by equating the inductively stored energy to the capacitive energy from which it is derived:

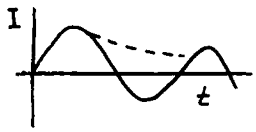

Fig. 7. Circuit ringing at natural frequency if not crowbarred.

$$\frac{1}{2}\ LI^2 = \frac{1}{2}\ CV^2; \quad I = V\ \frac{C}{L} = \frac{V}{N}\ \frac{C}{L_o} = \frac{40\ \text{kA}}{N} \qquad (13)$$

where L_o denotes the single-turn inductance of the drive coil. To determine the minimum number of turns for each drive coil, we require that the current reach its peak value no later than the time at which the bucket coil reaches the position of maximum propulsion efficiency, that is, the point of maximum dM/dx, which is 3.5 cm (Fig. 5). Equating one quarter-period to the travel time gives the condition for maximum N:

$$\frac{\pi}{2}\ \frac{1}{\omega} = \frac{.035}{u}\ ; \quad N_{max} = \frac{989}{u} \qquad (14)$$

At the nominal acceleration of 1,000 m/s^2, this indicates a maximum of 99 turns for the first coil, 66 for the second, and 16 for the twentieth.

The L/R time constant which governs the current decay after crobarring, and which depends only on the coil cross section, turns out to be 10 ms. It is obviously inefficient to allow the current to decay significantly while the bucket coil is still in a position to derive energy from this current, say within the 10 cm block following the drive coil. This condition is satisfied in all but the first three coils. In these, the bucket transit time is still of the order of 10 ms. We therefore enlarge the cross section of the first three coils. Taking into account these conditions, supplemented by some practical considerations (availability of square copper wire, SCRs and diodes) we chose the following coil parameters:

coil no.	turns	width	build	max I_d
1	80	6 cm	2 cm	500 A
2	64	5	2	625
3	64	5	2	625
4-10	32	2	2	1.25 kA
11-20	16	2	2	2.5

The construction of these coils is described in the companion paper[13].

7. Computation of Performance

Equations (8) to (11) can be expressed as finite difference equations and solved by numerical integration using a programmable pocket calculator. Programs for HP-67 and HP-25 calculators are given below. The procedure is to divide each block between neighboring drive coils into intervals Δx, each being traversed in a time interval Δt. Each computation step begins with initial values of all variables and ends with updated final values, which then serve as the initial values for the next step. The basic difference equations are:

$$\Delta i = [\ + \frac{V}{L_o N^2} - \frac{I_b u}{L_o N}\ \frac{dM}{dx} - \frac{R_o}{L_o}\ i\ ]\ \Delta t \qquad (15)$$

$$\Delta V = -\ \frac{i}{C}\ \Delta t \qquad (16)$$

$$\Delta t = \frac{\Delta x}{u} \qquad (17)$$

$$a = \frac{F}{m} = \frac{i\ I_b\ N}{m}\ \frac{dM}{dx} \qquad (18)$$

$$u = u_o + a\ \Delta t \qquad (19)$$

$$e = \frac{m\ u^2}{2} \qquad (20)$$

i = drive coil current, A
V = capacitor voltage, V
N = number of turns of drive coil
C = capacity, F
R_o = single turn resistance, ohm
L_o = single turn inductance, H
I_b = total bucket current, ampere-turns
M = mutual inductance, H
t = time, sec
u = velocity, m/s
a = acceleration, m/s^2
e = energy, J
F = thrust force, N
m = bucket mass, kg
Δx, Δt, ΔV, etc., computation step increments
x = displacement from dead center, m

O'Neill approximation function (Fig. 5):

$$\frac{dM}{dx} = a_2\ x\ e^{-a_3 (x_{abs} + 10^{-9})^{a_4}} \qquad (21)$$

where: $a_2 = 4.062 \times 10^{-5}$
$a_3 = 43.918$
$a_4 = 1.17$

(The very small term in the exponent serves to prevent zero, which causes an error condition in the HP-25 program).

8. HP-67 Program

Program Description

The program uses finite difference equations (15) to (20) to compute the motion of a mass driver bucket in finite steps, taking into account one bucket coil and one drive coil. Computation can begin at any x-position, positive or negative, where the SCR is triggered. The capacitor is automatically crowbarred when its voltage reaches zero. During each calculation step, results are displayed for one second in the following order:

capacitor voltage V, volts
drive coil current i, amps
acceleration a, m/s^2
velocity u, m/s
initial energy for step, e_o, joules
change in energy Δe, joules
final energy for step e_f, joules
final x-position for step, x, meter

The final x-value remains displayed for 5 seconds with flashing decimal point; computation of the next step then begins, using the updated variables as initial values.

The program will operate in two alternative modes:

Mode a (analytical) uses dM/dx values calculated from an approximation curve developed by O'Neill[16] based on three parameters (see Fig. 5). Any arbitrary starting position and any computation interval Δx can be used, and computation will continue until it is stopped manually. This mode is useful for determining exact shape of waveform to high precision using small x-intervals.

Mode b (tabular) uses dM/dx values obtained from Grover[15] tabulation, or measured inductance gradient values, at ten positions, 1 cm apart (x = 0, 0.01, 0.02,...,0.10 m). Starting position may be at any integral centimeter from -0.1 to +0.1 m, and computation interval is automatically set at 0.01 m. Computation stops automatically when last tabulated dM/dx value has been used, at 0.11 m.

Initialization

The following data are stored in the register indicated, either manually or from a data card.

Constants:

B	N+Δx	m *)
C	C	F
D	L_o	H
E	R_o	Ω
9	m	kg
8	I_b	A-turns

Variables:

0	i	A
1	V	V
2	x	m
3	u	m/s
4	a	m/s^2
5	Δt	s
6	e	J
7	dM/dx	H/m

dM/dx values, mode (b)

S0	dM/dx at x = 0	H/m
S1	0.01	
S2	0.02	
S3	0.03	
S4	0.04	
S5	0.05	
S6	0.06	
S7	0.07	
S8	0.08	
S9	0.09	
A	0.10	

dM/dx parameters, mode (a)

S2	a_2
S3	a_3
S4	a_4

parameters used in O'Neill approximation function, defined in Fig. 5.

*) since all registers are used, it was necessary to combine N (an integer) with x (a fraction) in a single register, and rely on the HP-67's ability to recall them separately. The sum has no mathematical significance.

Remarks: initial value of all variables must be stored, except for a, Δt, and dM/dx in reg. 7; these values are computed during each step before they are used. Initial values may be zero where applicable, except for the initial velocity u:

Limitation: Initial velocity u must not be zero since this will cause an error condition when program attempts to compute the first time interval. Use average velocity for first step when initializing the first block (about 5 m/s)

User Instructions

Step	Instructions	Keys	Output
1	Load sides 1,2		
2	Initialize, using either analytic or tabular dM/dx data		
3	Store non-zero initial velocity.	STO3	
4	Select analytic mode or	f a	0.000 x
5	Select tabular mode (Δx automatically set to 0.01)	f b	1.000 x
6	START in mode a, program will compute until stopped manually by pressing R/S; in mode b, program will stop when x = .11 and will display 21 (I-address of last dM/dx register)	A	V, volts i, amps a, m/s^2 u, m/s e_o, joule Δe, joule e_f, joule x_f, meter (5 sec) repeat above...
7	Initialize for computation of next block by storing V_o, i_o = 0, and x_o. All other variables carry over from previous block.		
8	To compute next block, go to step 6. To copy any output, press R/S when value is displayed during 1 sec pause. Program can also be stopped during 5 sec -x- flash pause at end of step and desired data recalled from appropriate register.	R/S	
9	To resume operation after manual stop, press R/S. Pressing A will return to beginning of program which requires re-initializing variables.		

HP-67 Program Listing

Following is a complete listing of the program with explanatory annotations. Listing was printed on an HP-97. A recorded program can be obtained by sending me a blank magnetic card. I will appreciate any comments from users of this program, by mail or telephone (617) 253 5554.

```
001  *LBLa   analytic mode
002   STO2   store x
003    SF0
004    CF1
005      0 } indicates
006    PSE } analyt mode
007    X⇄Y
008    R/S
009  *LBLb   tabular mode
010   STO2   store x
011    SF1
012    CF0
013   RCLB } change Δx to
014    INT | .01 (in case
015      . | other value
016      0 | was stored)
017      1 |
018      + |
019   STOB }
020      1 } indicates
021    PSE } tabular mode
022    CLX
023   RCL2   recall x to x
024    R/S
025  *LBLA
026    FIX }  format
027   DSP3 }
028    F0?
029   GSB0   calc dM/dx
030    F1?
031   GSB1   find dM/dx
032   RCL2
033   RCL6 }
034   RCL3 |
035      x |
036   RCLD |      I_B u   dM
037   RCLB |  -  ------  ----
038    INT |      L_o N   dx
039      x |
040      ÷ |
041   RCL7 |
042      x |
043    CHS }
044   ENT↑ }
045   RCLE |      R_o
046   RCLD |  -  ----- i
047      ÷ |      L_o
048   RCL0 |
049      x |
050      - }
051   RCL1 }
052   X<0? |        V
053    CLX |  +  -------
054    PSE |     L_o N^2
055   RCLD |
056   RCLB |  zero if
057    INT |  V negative
058     X² |  (crowbar)
059      x |
060      ÷ }
061      +       [  ]
062   RCLB }  Δx
063    FRC }
064   RCL3    u
065      ÷    Δt
066   STO5   update Δt
067      x
068   ST+0   update i
069   RCL0 }
070    PSE } display new i
071   RCL5   Δt
072      x
073   RCLC   C
074      ÷   ΔV
075   ST-1   update V
076   RCL0 }
077   RCL8 |
078      x |
079   RCLB |  i I_B N   dM
080    INT |  -------  ----
081      x |     m      dx
082   RCL9 |
083      ÷ |  = accel.
084   RCL7 |
085      x }
086   STO4
087    PSE   display acc.
088   RCL5
089      x
090   ST+3   update u
091   RCL3 }
092    PSE } display u
093     X²
094   RCL9
095      x
096      2
097      ÷   new e
098   ENT↑
099   ENT↑
100   RCL6   old e
101    PSE   display old e
102      -
103    PSE   display Δe
104     R↓
105   STO6
106    PSE   display new e
107   RCL3
108   RCLB }
109    FRC }  Δx
110   ST+2   update x
111    X⇄Y
112      ÷   Δt
113   STO5   update Δt
114   RCL2
115   PRTX   -x- end of cyc.
116   GTOA   repeat cycle
117  *LBL0   routine cal-
118    P⇄S   culates dM/dx
119   STO1   from o'Neill
120   RCL2   function,
121      x   starts with
122   RCL1   x in x
123    ABS
124    EEX   prevents x=0
125      9
126    CHS
127      +
128   RCL4   a_4
129     Yˣ
130   RCL3   a_3
131      x
132    CHS
133     eˣ
134      x   =dM/dx, O'Neill
135    P⇄S
136   STO7   update dM/dx
137    RTN
138  *LBL1   routine finds
139   X<0?   tabular dM/dx
140    SF2   uses correct
141    ABS   sign
142    EEX   convert x to cm
143      2
144      x
145      1   add 10 to |x|
146      0   for use to in-
147      +   crement I reg.
148   STOI
149      2
150      1
151   X=Y?   end of table ?
152    R/S   STOP
153   RCLi   tab value
154    F2?   chs if x neg
155    CHS
156   STO7   update dM/dx
157    RTN
158    R/S
```

Data for analytic mode		Data for tabular mode	
PREG		PREG	
0.000	0	0.000	0
450.000	1	450.000	1
-0.010	2	0.000	2
10.000	3	10.000	3
0.000	4	0.000	4
0.000	5	0.000	5
0.000	6	0.000	6
0.000	7	0.000	7
50000.000	8	50000.000	8
0.500	9	0.500	9
0.000	A	2.255000000-07	A
80.010	B	80.010	B
0.002	C	0.002	C
2.544000000-07	D	2.544000000-07	D
2.432000000-05	E	2.432000000-05	E
0.000	I	11.000	I
P⇄S PREG		P⇄S PREG	
0.000	0	0.000	0
-0.010	1	1.648000000-07	1
4.362000000-05	2	4.379000000-07	2
43.918	3	5.741000000-07	3
1.170	4	5.955000000-07	4
0.000	5	5.517000000-07	5
0.000	6	4.796000000-07	6
0.000	7	4.043000000-07	7
0.000	8	3.353000000-07	8
0.000	9	2.755000000-07	9
0.000	A	2.259000000-07	A
80.010	B	80.010	B
0.002	C	0.002	C
2.544000000-07	D	2.544000000-07	D
2.432000000-05	E	2.432000000-05	E
0.000	I	11.000	I

Results

The HP-67 program was used to compute the performance of the basic mass driver under construction, with the initializing data for tabular mode listed above. The twenty coils had the turns indicated, but the first three coils had not been enlarged, i.e., all coils had the same cross section of 4 cm^2 and the same L/R time constant. The SCR in each block was triggered at x = -.01m, i.e., when the bucket coil was 1 cm before dead center. This corresponds to advancing the spark of a gasoline engine to fire before top dead center so as to provide more time for pressure to build up. The slight negative acceleration which results is more than offset by the higher current (pressure) following dead center. An exact optimization was not made as yet.

Computation requires 12.5 sec/step, and 3.5 min for each 11-step block, including pause between steps. Total computation of all 20 blocks takes 70 minutes.

At the end of the 2 m accelerating section the bucket has a kinetic energy of 1308 J, about 30% of the 4050 J stored in all the capacitors. Average acceleration is 1186 m/s^2 and final velocity is 72.3 m/s (162 mph). It is interesting to note that the highest current peak occurs in block 11 rather than block 20 as one might expect.

The braking effect of a short-circuited coil can be computed by simply setting the initial capacitor voltage to zero. Deceleration depends only on the L/R time constant, which in turn depends only on the cross section (resistance) of the coil, and not on the number of turns. The braking coils can thus be simple aluminum rings of optimized cross section. As expected, the optimized cross section is obtained by equating the electrical and mechanical time constants:

$$\frac{\lambda}{u} = \frac{L}{R} \qquad (21)$$

where u is velocity and λ is the characteristic decay length of the dM/dx curve in Fig. 5, say .05 m. Unfortunately a passive electric brake of this type can extract only about 9 J per ring at the maximum velocity, and decreasing amounts as the bucket slows down; this is not a practical means to dissipate a kilojoule of energy. A friction brake is therefore being constructed, as described in the companion paper[13].

9. HP-25 Program

It is also possible to calculate the performance of a basic mass driver using an HP-25 calculator, although considerable ingenuity is required to operate within the limitation of 50 program steps and only 8 registers. For the benefit of HP-25 owners, we present here a program developed by Professor Gerard K. O'Neill[16], who finds it very difficult to resist a challenge of this sort.

```
00  AUTOMATIC STOP
01  ST-6   CROWBARS ON V<0
-------------------------------------
02  .   Δx = .0n1n2 START         x subroutine
03  0                             n1=1, n2=2 for
04  n1                            .01 meter steps
05  n2                            reg. 1 and x are
06  STO+1                         incremented
07  RCL1   x+Δx END
-------------------------------------
08  RCL2   a2 = -44.00004062
09  FRAC   x = -4.062 10^-5
10  x      = a2F x
11  RCL1   x                      (dM/dx)subroutine
12  ABS    |x|                    15 steps, starts
13  1                             with x in x and in
14  .                             reg. 1;
15  1                             ends with -(dM/dx)
16  7                             x=0 is illegal
17  y^x
18  RCL2
19  INT    x= - 44.0
20  x      (-44.0)x_ABS
21  e^x
22  x   -(dM/dx)   END
-------------------------------------
23  ↑   starts with dM/dx   START
24  ↑    y=z= - dM/dx             , difference equa-
25  RCL7   x=i                    - tion subroutine
26  x                              22 steps
27  ST-5 update + Σ(i)(dM/dx)
28  ↑↓
29  RCL0   +a0
30  x    -a0(dM/dx)
31  RCL6     V
32  α1
33  α2
34  α3   x=α Y=V,    =-a0(dM/dx)
35  x
36  +    x = αV - a0(dM/dx)
37  RCL3   a3
38  RCL7   i
39  x      a3i
40  -   Δi = αV - a0(dM/dx) - a3i
41  ST+7  update k. x = Δi
42  RCL7   new i
43  RCL4   a4
44  x                       END
-------------------------------------
45  ST-6
46  RCL6       update V
-------------------------------------
47  x<0             SCR switching and diode crowbar
48  GTO 01          subroutine 47-01
49  GTO 02
-------------------------------------
```

REGISTERS:

0 $a_o = (\Delta x) i_B / L_o N$

1 x^o

2 a_2: INT = -44 FRAC = - 4.062×10^{-5}

3 $a_3 = (\Delta x) R / u L_o N^2$

4 $a_4 = (\Delta x)/uC$

5 $\Sigma(i)(dM/dx)$

6 V

7 i

$\alpha_1\ \alpha_2\ \alpha_3 = (\Delta x)/uL_oN^2$ (incl. decimal)

$\Delta i = \alpha V - a_o(dM/dx) - a_3 i$

$\Delta V = -a_4 i \qquad u = dx/dt$

$x_n = x_{n-1} + .0n_1n_2$ (x=0 not allowed)

INITIALIZATION:

$.0n_1n_2$ = interval in meters

usually $.0n_1n_2$ = .005; stored in prgm 02-05

R1 for x_{in} = -1cm, set R1 = -.0101 to avoid x=0
R2 stores two constants used in (dM/dx) routine
R3 winding resistance R appears only here
R4 capacity C
R5 set = 0 to start
R6 normally 450 volts
R7 set = 0 to start

$\alpha_1\ \alpha_2\ \alpha_3$ three digits incl. decimal, typically $0.23' < \alpha < 1.1$

CROWBAR SUBROUTINE (steps 47,48,49,01) when capacitor voltage falls below zero, goes to step 01, ST-6, which forces V=0

RUNNING TIME: 3.75 sec for entire program. Timing 90 seconds gives an integration of 12 cm (24 steps) Registers R5, R6, R7, R1 must be initialized on each run; normally terminate run at 90 sec by R/S.

DIAGNOSTIC:

To trace the current and voltage waveforms, make the program change at right:

```
43  STOP, look at i
44  RCL4
45  x
46  ST-6
47  RCL6
48  STOP  look at V
```

when V<0, change 49 from GTO 02 to GTO 01.

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DISCUSSION

Q. Have you considered what would happen to the stabilization of the superconducting coils due to fluctuations in the field?

A. We faced that problem in the case of the Magneplane. We were able to forcibly demobilize the Magneplane on its guideway, during full acceleration of the motor, without quenching the superconductivity. A simple aluminum shield placed around the superconductor shields the vehicle from fluctating fields in its frame of reference. Of course, there are higher harmonics in the drive circuits so the vehicle will not see pure DC.

Q. As the payload is released and the buckets decelerate, doesn't the friction on the soil undo all the sensitive aiming?

A. Only part of the aiming is done before the payload is released. After the load is released, there will be forces on it due not only to the friction but also to the fact that the payload is likely to be slightly paramagnetic. Therefore the payload motion will be sensed after it leaves the bucket and corrected by electrostatic deflection plates in a manner which is described later by O'Neill and in two technical papers from the 1976 Ames Summer Study (in Progress in Aeronautics & Astronautics 1977). The nice thing about this orbit calculation is the fact that if you wait long enough the secular departure from the orbit will become as large as you please, so that millimeters per second accuracy at launch turns into many centimeters some kilometers downrange, which is very easy to correct. It may not be so easy to correct at the instant of release.

Q. Could you give some figure for the ultimate exhaust velocity of a mass driver system when used as a production system?

A. There is no inherent limit to the velocity of a mass driver, except, of course, the speed of light. The ultimate limit in acceleration is imposed by what superconductors can stand. Presently available superconductors can carry 2.5×10^4 amperes-cm^{-2} and can withstand a field of 100 kilogauss: this translates into an acceleration of 0.5×10^5 gravities.

Q. What limits the velocity? Is it the mechanical system?

A. The acceleration is limited by the strength of the structure because you're not going to accelerate only a superconducting loop; you want to attach something to it.

Size, of course, is a key factor. I cite the well-known case of the horse versus the flea: strength does not scale very simply with size. There's also the question of the exact structure to be handled. The ultimate acceleration is one of the things we hope to tackle in the 1977 Summer Study; it's a very important question. To answer your question, there is no limit to the ultimate speed.

It's interesting to reflect upon the fact that a deuterium ion going 10 km-s^{-1} has an energy of 1 eV. It's generally accepted that to achieve fusion you need an energy of 10^4 eV per ion. So we're within four orders of magnitude of achieving fusion by simply using a mass driver to accelerate a pellet of deuterium into a brick wall.

Q. I would like to comment on this problem of the accuracy. The optimal accuracies, it now appears, can be defined by means of engineering trade studies in which the mass catcher and the fine guidance system of the mass driver are studied together and designed together as an integrated system. I believe it will be possible to define this problem within a matter of weeks or perhaps a few months, and to study the problem in satisfactory fashion next year.

A. (None)

Q. The scaling laws for the masses of the mass driver as a function of the acceleration indicate that the acceleration is likely to be limited not by the inherent strength of materials but by the fact that you simply have to pay more in terms of mass as the acceleration goes up. That is, we're easily able to get into the range of 10 km-s^{-1} in exhaust velocity. Twenty km-s^{-1} gets to the point where just about everything escapes, and Davis has already suggested that maybe that's the way to go. You just pay some mass for it.

A. (None)