Reinventing Space:
Low-Cost, Responsive Space Missions
USC ASTE 523, Spring 2021

Supplement 1

Money

A. Inflation

B. The Time Value of Money

C. Amortization

D. Learning Curves

INFLATION

- ***Inflation*** **represents the decreasing value of money over time**
  - **Assume all the world is an island with a single ruler and a single currency**
  - **The beneficent ruler, being both beneficent and not too bright, decides to give everyone on the island a pay raise and doubles their salary**
  - **Since it takes the same amount of labor to produce the islands goods and services, the value of island dollar is halved and we have 100% inflation**
  - **Similarly *deflation* occurs when the value of money increases with time**
- **Inflation and deflation are not the same as true economic growth or real cost reductions, based on being able to do more at less cost**
  - **Computers are able to do more and cost less over time due to real economies of scale and production efficiency — i.e., it takes less capital and labor to build a computer with a given capacity than it did to build that same capacity 20 years ago**
  - **In an average sense, inflation represents that portion of increases in wages and other production costs not offset by improvements in efficiency**
- **Inflation rates estimated by the government (See SME Table 11-31)**
  - **Use inflation rates to bring all costs to *constant year dollars***
    - **Example: *FY10$M* = millions of Fiscal Year 2010 dollars**
  - **We will typically ignore inflation by using constant year dollars**

INFLATION FACTORS RELATIVE TO THE YEAR 2012

Fiscal Year	Inflation Factor to Base Year 2012	Fiscal Year	Inflation Factor to Base Year 2012
1995	0.6408	2016	1.0857
1996	0.6558	2017	1.1089
1997	0.6723	2018	1.1326
1998	0.6913	2019	1.1568
1999	0.7102	2020	1.1815
2000	0.7285	2021	1.2067
2001	0.7516	2022	1.2324
2002	0.7717	2023	1.2588
2003	0.7979	2024	1.2856
2004	0.8240	2025	1.3131
2005	0.8583	2026	1.3411
2006	0.8867	2027	1.3698
2007	0.9108	2028	1.3990
2008	0.9375	2029	1.4289
2009	0.9501	2030	1.4594
2010	0.9701	2031	1.4906
2011	0.9850	2032	1.5224
2012	1.0000	2033	1.5549
2013	1.0201	2034	1.5881
2014	1.0413	2035	1.6221
2015	1.0630	2036	1.6567

Beyond 2012, the rates shown are based on an extrapolated constant rate of inflation. New projections are available annually.

THE TIME VALUE OF MONEY

- **Like most things, money isn't free**
- **Even ignoring inflation, spending $1000 now costs more than spending $1000 next year, and much more than spending $1000 ten years from now**
  - **If we don't have the money we will have to borrow it and pay interest**
  - **If we do have the money, we will have to forego using it for some alternative purpose capable of generating interest or income**
  - **Mathematically the two approaches are the same and represent the time value of money**
- ***Future value of money* (FV) = what a given amount of money (the *present value,* PV) would be worth at some time in the future**

$$FV = PV\,(1 + i)^N$$

where i is the interest rate per period and N is the number of future periods

- **In the limit of continuous compounding, the *effective interest rate,* EI, is:**

$$EI = e^i - 1$$

$$FV = PV\,(1 + EI)^Y$$

where Y is the number of years and i is the nominal annual interest rate

EXAMPLES OF FUTURE VALUE

- Assume 10% annual interest (compounded annually)
  - — $1000 now is worth $1,100 in 1 year
  - — Worth $1000 x 1.1^{10} = $2,593.74 in 10 years
  - — Expressed as the inverse problem:
    - Spending $1000 now costs $1000 now
    - Spending $1000 1 year from now costs $909.09 now
    - Spending $1000 10 years from now costs $385.54 now

- Same example with continuous compounding
  - — Effective interest rate = 10.517%
  - — $1000 now is worth $1,105.17 in 1 year
  - — Worth 1000 x 1.10517^{10} = $2,718.28 in 10 years
  - — Spending $1,000 10 years from now costs $367.88 now

- **The future value of money depends mathematically on how often interest is paid, but the differences are not great and can be ignored for most system design purposes**
  - — Errors will be dominated by other issues

Putting off spending reduces the real cost. However, schedule delays or increased operations costs may more than offset these savings.

AMORTIZATION

- **The time value of money is represented by the interest rate that I have to pay to borrow it (or, equivalently, the interest I can get by lending it)**
  - — ***Amortization*** **is the process of paying off over time**
  - — **Normally we amortize a loan by making a fixed number of equal payments**
    - **Each payment first covers the interest due, with the remainder applied to the *principal* (i.e., the remaining amount to be repaid)**

- **Value of N equal payments at interest i (per payment period) is:**

$$\text{Payment} = PV \times [\, i / (1 - (1 + i)^{-N}) \,]$$

where PV is the amount of the loan

$$\text{Total cost} = N \times \text{Payment}$$

$$\text{Total interest expense} = \text{Total cost} - PV$$

- **An *amortization schedule* is a tabular listing of the interest, principal, and balance due on the loan for each payment period**

AMORTIZATION EXAMPLE

- Assume that we can reduce operations cost by spending $5 million on an autonomous system and that we want to recover (i.e., pay off) the required investment equally over a 10 year mission life with an assumed interest rate of 9%
  - Payment = $5{,}000{,}000 \times [\ .09 / (1 - (1.09)^{-10})\ ] = \$779{,}100.45$
  - Amortization schedule:

Step	Payment	Interest	Principal	Balance
0				$5,000,000.00
1	$779,100.45	$450,000.00	$329,100.45	$4,670,899.55
2	$779,100.45	$420,380.96	$358,719.49	$4,312,180.06
3	$779,100.45	$388,096.21	$391,004.24	$3,921,175.82
4	$779,100.45	$352,905.82	$426,194.63	$3,494,981.19
5	$779,100.45	$314,548.31	$464,552.14	$3,030,429.05
6	$779,100.45	$272,738.61	$506,361.84	$2,524,067.21
7	$779,100.45	$227,166.05	$551,934.40	$1,972,132.81
8	$779,100.45	$177,491.95	$601,608.50	$1,370,524.32
9	$779,100.45	$123,347.19	$655,753.26	$714,771.05
10	$779,100.45	$64,329.39	$714,771.05	$0.00
Totals	$7,791,004.50	$2,791,004.50	$5,000,000.00	

- Need to reduce ops cost by more than $779K/year to make the expenditure worthwhile

LEARNING CURVES

- *Learning curves* express the fact that building 2 of something costs more than building 1, but not twice as much
- Definitions
  — *Theoretical first unit* (TFU) = cost of a first production unit if only one is built
  — *Learning curve slope = learning curve percent* (S) = percentage reduction in cumulative average cost when the number produced is doubled (spoken as “95% learning curve”)
    - If the TFU cost is $1000 and a 95% learning curve applies, the second unit will cost $900, such that the average cost of the first two units will be $950
    - In this example, the average cost of the first 4 units will be $1000 x 95% x 95% = $902 and the average cost of the first 8 units will be $902 x 95% = $857
- Total production cost for N units with a fixed learning curve, S, is:

$$\text{Total production cost} = \text{TFU} \times L$$

$$L = N^B$$

$$B = 1 - [\, \log(100\% / S) / \log 2 \,]$$

- Example: If S = 95%, B = 0.926 and the cost of 100 units = $100^{0.926}$ = 71.1 x the cost of a single unit (for a 90% learning curve, 100 units = 49.7 x cost of first unit)

APPLYING LEARNING CURVES

- **Typical Learning Curve Values:**

Less than 10 units	95%
10 to 50 units	90%
More than 50 units	85%

- **Representative Results**

	S = 95%		S = 90%		S = 85%	
N	Average cost	Nth unit cost	Average cost	Nth unit cost	Average cost	Nth unit cost
1	1,000	1,000	1,000	1,000	1,000	1,000
10	843	784	705	602	583	452
100	711	659	497	421	340	260
1,000	600	555	350	297	198	152
10,000	506	468	247	209	115	88

CONSEQUENCES OF LEARNING CURVE

- **Assume a TFU cost of $50 million per satellite**
- **For a 50 satellite constellation ($2.5 billion without learning curve)**

	90% LC	85% LC
— Total satellite cost	$1.38 B	$1.00 B
— Average cost per satellite	$27.6 M	$20.0 M
— Last satellite cost	$23.4 M	$15.4 M
— Cost of last 5 satellites	$117 M	$77 M

- **For a 100 satellite constellation ($5 billion without learning curve)**

	90% LC	85% LC
— Total satellite cost	$2.48 B	$1.70 B
— Average cost per satellite	$24.8 M	$17.0 M
— Last satellite cost	$21.1 M	$13.0 M
— Cost of last 5 satellites	$106 M	$65 M

The learning curve dramatically reduces the cost of the constellation, and also reduces the savings from reducing the number of satellites.